NAG Toolbox for MATLAB

d05ab

1 Purpose

d05ab solves any linear nonsingular Fredholm integral equation of the second kind with a smooth kernel.

2 Syntax

```
[f, c, ifail] = d05ab(k, g, lambda, a, b, odorev, ev, n)
```

3 Description

d05ab uses the method of El–Gendi 1969 to solve an integral equation of the form

$$f(x) - \lambda \int_a^b k(x,s) f(s)\, ds = g(x)$$

for the function $f(x)$ in the range $a \leq x \leq b$.

An approximation to the solution $f(x)$ is found in the form of an n term Chebyshev-series $\sum_{i=1}^{n}{}' c_i T_i(x)$, where $'$ indicates that the first term is halved in the sum. The coefficients c_i, for $i = 1, 2, \ldots, n$, of this series are determined directly from approximate values f_i, for $i = 1, 2, \ldots, n$, of the function $f(x)$ at the first n of a set of $m + 1$ Chebyshev points

$$x_i = \tfrac{1}{2}(a + b + (b - a) \times \cos[(i - 1) \times \pi/m]), \qquad i = 1, 2, \ldots, m + 1.$$

The values f_i are obtained by solving a set of simultaneous linear algebraic equations formed by applying a quadrature formula (equivalent to the scheme of Clenshaw and Curtis 1960) to the integral equation at each of the above points.

In general $m = n - 1$. However, advantage may be taken of any prior knowledge of the symmetry of $f(x)$. Thus if $f(x)$ is symmetric (i.e., even) about the mid-point of the range (a, b), it may be approximated by an even Chebyshev-series with $m = 2n - 1$. Similarly, if $f(x)$ is anti-symmetric (i.e., odd) about the mid-point of the range of integration, it may be approximated by an odd Chebyshev-series with $m = 2n$.

4 References

Clenshaw C W and Curtis A R 1960 A method for numerical integration on an automatic computer *Numer. Math.* **2** 197–205

El–Gendi S E 1969 Chebyshev solution of differential, integral and integro-differential equations *Comput. J.* **12** 282–287

5 Parameters

5.1 Compulsory Input Parameters

1: **k – string containing name of m-file**

k must compute the value of the kernel $k(x, s)$ of the integral equation over the square $a \leq x \leq b$, $a \leq s \leq b$.

Its specification is:

```
[result] = k(x, s)
```

Input Parameters

1: **x – double scalar**

2: **s – double scalar**

The values of x and s at which $k(x,s)$ is to be calculated.

Output Parameters

1: **result – double scalar**

The result of the function.

2: **g – string containing name of m-file**

g must compute the value of the function $g(x)$ of the integral equation in the interval $a \le x \le b$.

Its specification is:

```
[result] = g(x)
```

Input Parameters

1: **x – double scalar**

The value of x at which $g(x)$ is to be calculated.

Output Parameters

1: **result – double scalar**

The result of the function.

3: **lambda – double scalar**

The value of the parameter λ of the integral equation.

4: **a – double scalar**

a, the lower limit of integration.

5: **b – double scalar**

b, the upper limit of integration.

Constraint: **b** $>$ **a**.

6: **odorev – logical scalar**

Indicates whether it is known that the solution $f(x)$ is odd or even about the mid-point of the range of integration. If **odorev** is **true** then an odd or even solution is sought depending upon the value of **ev**.

7: **ev – logical scalar**

Is ignored if **odorev** is **false** Otherwise, if **ev** is **true**, an even solution is sought, whilst if **ev** is **false**, an odd solution is sought.

8: **n – int32 scalar**

the number of terms in the Chebyshev-series which approximates the solution $f(x)$.

Constraint: **n** $>$ 0.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

cm, f1, wk, ldcm, nt2p1

5.4 Output Parameters

1: **f(n) – double array**

The approximate values f_i, for $i = 1, 2, \ldots, \mathbf{n}$, of the function $f(x)$ at the first **n** of $m + 1$ Chebyshev points (see Section 3), where

$m = 2\mathbf{n} - 1$
if **odorev** $=$ **true** and **ev** $=$ **true**

$m = 2\mathbf{n}$ if **odorev** $=$ **true** and **ev** $=$ **false**

$m = \mathbf{n} - 1$
if **odorev** $=$ **false**

If **odorev** is **true**, then $m = 2 \times \mathbf{n} - 1$ if **ev** is **true** and $m = 2 \times \mathbf{n}$ if **ev** is **false**; otherwise $m = \mathbf{n} - 1$.

2: **c(n) – double array**

The coefficients c_i, for $i = 1, 2, \ldots, \mathbf{n}$, of the Chebyshev-series approximation to $f(x)$. When **odorev** is **true**, this series contains polynomials of even order only or of odd order only, according to **ev** being **true** or **false** respectively.

3: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, $\mathbf{a} \geq \mathbf{b}$ or $\mathbf{n} < 1$.

ifail $= 2$

A failure has occurred due to proximity to an eigenvalue. In general, if **lambda** is near an eigenvalue of the integral equation, the corresponding matrix will be nearly singular. In the special case, $m = 1$, the matrix reduces to a zero-valued number.

7 Accuracy

No explicit error estimate is provided by the function but it is possible to obtain a good indication of the accuracy of the solution either

(i) by examining the size of the later Chebyshev coefficients c_i, or

(ii) by comparing the coefficients c_i or the function values f_i for two or more values of **n**.

8 Further Comments

The time taken by d05ab depends upon the value of **n** and upon the complexity of the kernel function $k(x, s)$.

9 Example

d05ab_g.m

```
function [result] = g(x)
  result = 1;
```

d05ab_k.m

```
function [result] = k(x, s)
  result = 1/(1+(x-s)*(x-s));
```

```
lambda = -0.3183;
a = -1;
b = 1;
odorev = true;
ev = true;
n = int32(5);
[f, c, ifail] = d05ab('d05ab_k', 'd05ab_g', lambda, a, b, odorev, ev, n)
```

```
f =
    0.7557
    0.7453
    0.7173
    0.6832
    0.6605
c =
    1.4152
    0.0494
   -0.0010
   -0.0002
    0.0000
ifail =
           0
```